

Calcul stochastique

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Cours : 24 heures - TP : 16 heures

Objectif

The course presents the basic material of a second level of stochastic calculus.

Plan

- Motivations: stochastic modeling, probabilistic representations of PDEs, stochastic control, filtering, mathematical finance.
- Stochastic processes in continuous time: Gaussian processes, Brownian motion, (local) martingales, semimartingales, Itô processes.
- Itô and chain rule formulae, a first approach to stochastic differential equations.
- Girsanov formulae, predictable representation of Brownian martingales.
- Stochastic differential equations with Lipschitz coefficients. Markov flows.
- Stochastic differential equations without Lipschitz coefficients: Bessel type processes, existence and uniqueness in law.
- Connections with PDEs of parabolic and elliptic type.
- Elements of McKean-Vlasov type equations.
- Elements of backward stochastic differential equations.
- Basic connections with mathematical finance.

Close related references to the course are the following monographs and articles: [3, 7, 4, 6]. For deeper considerations, we also refer to [8, 9, 2, 5, 1].

Références

- [1] P. Blanchard, M. Röckner, and F. Russo. Probabilistic representation for solutions of a porous media type equation. *Ann. Probab.*, 38(5):1870–1900, 2010.
- [2] J. Jacod and A. N. Shiryaev. Limit theorems for stochastic processes, volume 288 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, second edition, 2003.
- Ioannis Karatzas and Steven E. Shreve. *Brownian motion and stochastic calculus*, volume 113 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1991.
- [4] Damien Lambert and Bernard Lapeyre. *Introduction au calcul stochastique appliqué à la finance*. Ellipses Édition Marketing, Paris, second edition, 1997.
- [5] David Nualart. *The Malliavin calculus and related topics*. Probability and its Applications (New York). Springer-Verlag, New York, 1995.
- [6] Étienne Pardoux. Backward stochastic differential equations and viscosity solutions of systems of semilinear parabolic and elliptic PDEs of second order. In *Stochastic analysis and related topics*, VI (Geilo, 1996), volume 42 of *Progr. Probab.*, pages 79–127. Birkhäuser Boston, Boston, MA, 1998.
- [7] Daniel Revuz and Marc Yor. *Continuous martingales and Brownian motion*, volume 293 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, third edition, 1999.
- [8] L. C. G. Rogers and David Williams. *Diffusions, Markov processes, and martingales*. Vol. 2. Cambridge Mathematical Library. Cambridge University Press, Cambridge, 2000. Itô calculus, Reprint of the second (1994) edition.
- [9] Daniel W. Stroock and S. R. Srinivasa Varadhan. *Multidimensional diffusion processes*. Classics in Mathematics. Springer-Verlag, Berlin, 2006. Reprint of the 1997 edition.