

Copulas and financial applications

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Course : 18 hours - TP : 0 hours

Objectives

This course will study the dependence between random variables through the concept of copulas. These probabilistic tools, defined in the 1950s, were rediscovered a few years ago, and have since given rise to an abundant scientific production. Practitioners themselves cannot ignore copulas, as multivariate modelling is now at the heart of financial issues. But because of their generality, copulas are a tool that can be applied in many other fields, including biology, medicine, reliability, etc. We will introduce some concepts and measures of dependency, the main families of copula and some associated probabilistic properties. We will focus on statistical inference of copula. Finally, we will study the applications of copulas to model joint risks, whether they are times of default or returns on assets.

Outline

- Generalities on copula - Basic definitions: density, survival copula, Sklar's theorem. Fréchet's terminals. Classical copula families (Gaussian, Student, Archimedean, Marshall-Olkin, etc). Methods for generating new copula. Models of factor copula. Dependency measures: tail indicators, Kendall's tau, etc.
- Statistical analysis of Copulas - Parametric and semi-parametric estimation. Non-parametric estimation, empirical copula. Choice of the right copula. Simulation of copula. Miscellaneous problems : independence test, bootstrap etc.
- Applications in risk management-Dependence between risk factors. Dependence between default times and asset values. Granular risks, operational risks
- Applications in Valuation-Valuation of CDOs. Exotic options on multiple underlyings.

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