Levy processes and applications in finance

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Course : 18 hours

Objectives

Theoretical and empirical studies show that for the valuation of options and especially for risk management, it is essential to take into account the possibility of a quasi-instantaneous movement of large amplitude (jump) in the price of assets. Levy processes are a class of processes with jumps that are both rich enough to describe the reality of the markets and simple enough to allow for rigorous processing and explicit calculations.

In the first part of this course, we will give a simplified mathematical introduction to Levy processes, Poisson random measures, which are the building blocks of Levy processes, and the bases of stochastic calculus for discontinuous processes.

The second part will focus on the financial applications of the Levy processes. We will not only discuss the theory of option pricing in Levy models, which is already well established in the literature, but also more recent topics such as risk management with Levy processes.

Outline

- Introduction: Motivations for using discrete processes in financial modeling; examples of Levy processes and discrete processes in general.
- Poisson process and composite Poisson process. Random Poisson measurements. Characteristic functions. Compound Poisson process simulation. Examples: Kou model, Merton model.
- Definition of a Levy process and examples of Levy processes of infinite jump intensity. Gamma process and gamma variance model.
- Measurement of jumps and Levy measurement of a Levy process. Characteristic function of a Levy process: Levy-Khintchine formula.
- Stochastic integrals with respect to random Poisson measurements. Quadratic variation and Itô's formula. Isometric relation for stochastic integrals. Doléans exponential.
- Exponential-Levy models. Measurement changes for Levy processes and absence of arbitrage in exponential-Levy models. Market incompleteness.
- Incomplete market coverage methods. Quadratic coverage in models with jumps.
- European options in the exp-Lévy models. Option valuation in exp-Lévy models by Fourier transform..

Bibliography

R. Cont and P. Tankov, Financial Modelling with Jump Processes, Chapman & Hall, CRC Press, 2004.