

Stochastic calculus

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Course : 32 hours - TP : 8 hours

Objectives

The course presents the basic material of a second level of stochastic calculus.

Outline

- Motivations: stochastic modeling, probabilistic representations of PDEs, stochastic control, filtering, mathematical finance.
- Stochastic processes in continuous time: Gaussian processes, Brownian motion, (local) martingales, semimartingales, Itô processes.
- Itô and chain rule formulae, a first approach to stochastic differential equations.
- Girsanov formulae, predictable representation of Brownian martingales.
- Stochastic differential equations with Lipschitz coefficients. Markov flows.
- Stochastic differential equations without Lipschitz coefficients: Bessel type processes, existence and uniqueness in law.
- Connections with PDEs of parabolic and elliptic type.
- Elements of McKean-Vlasov type equations.
- Elements of backward stochastic differential equations.
- Basic connections with mathematical finance.

Close related references to the course are the following monographs and articles: [3, 7, 4, 6]. For deeper considerations, we also refer to [8, 9, 2, 5, 1].

Bibliography

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- [4] Damien Lambert and Bernard Lapeyre. *Introduction au calcul stochastique appliqué à la finance*. Ellipses Édition Marketing, Paris, second edition, 1997.
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- [7] Daniel Revuz and Marc Yor. *Continuous martingales and Brownian motion*, volume 293 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, third edition, 1999.
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- [9] Daniel W. Stroock and S. R. Srinivasa Varadhan. *Multidimensional diffusion processes*. Classics in Mathematics. Springer-Verlag, Berlin, 2006. Reprint of the 1997 edition.